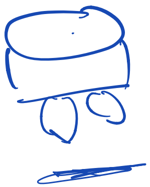
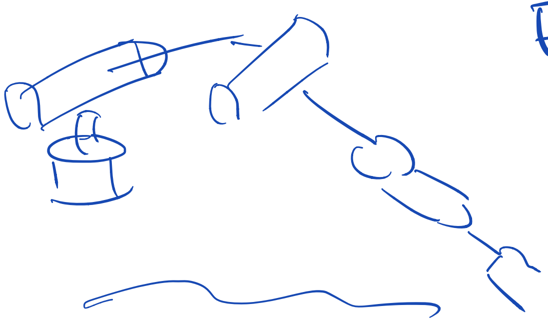


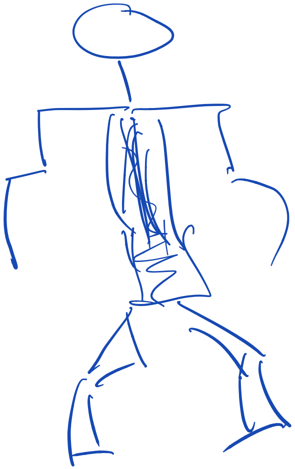


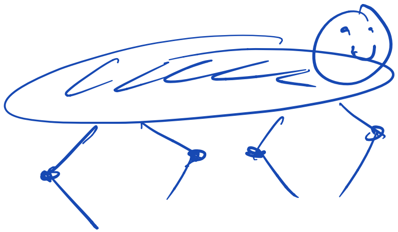
First Question:

How to specify
"where is the robot?"

→ Formalize this

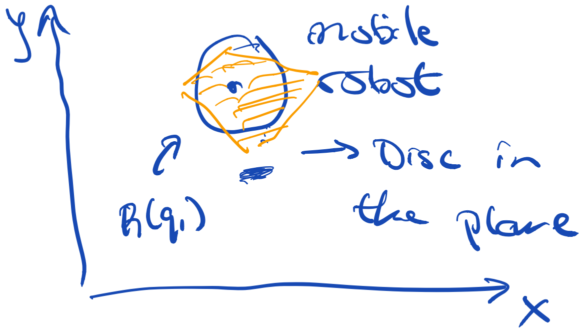
⇒ configuration

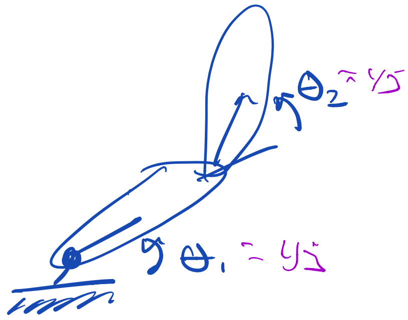




Definition: A configuration, q, is a complete spec. of the position of every point on the robot.

Def: The configuration space is the set of all configurations: Q

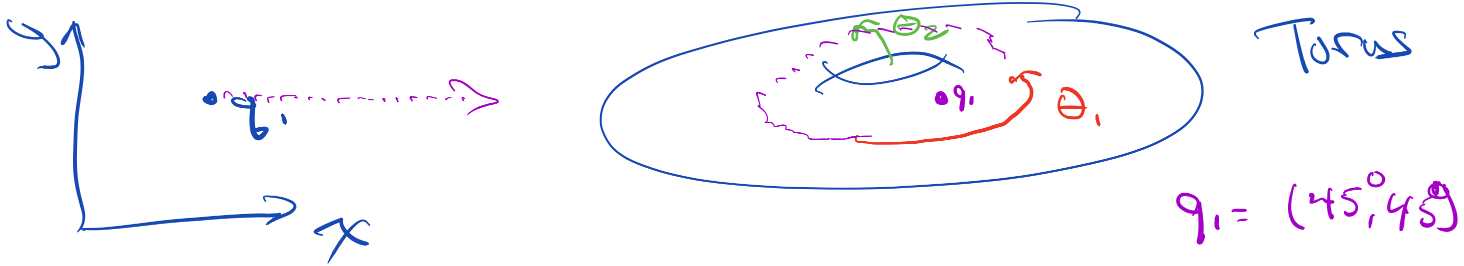




two_link robot arm, planar

Work space

Q Disc $= \mathbb{R}^2$ Q 2-link arm

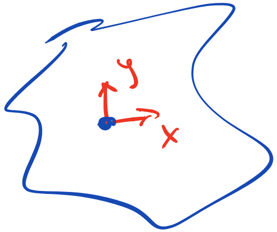


Let's start with rigid objects.

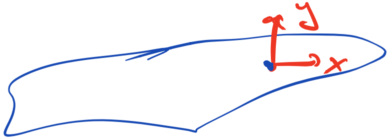
To specify configuration of a rigid object

1. Attach a coordinate frame
2. specify position + orientation of frame

In the plane:



Frame is rigidly attached.



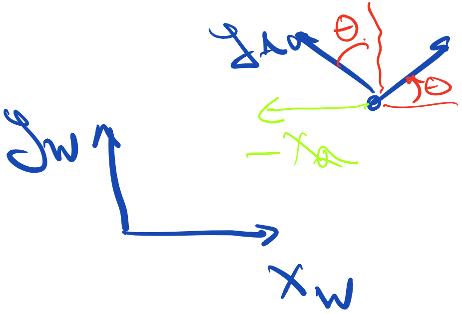
Assume we have a model of the robot.

How to specify position & orientation ??

position: $x, y \in \mathbb{R}^2$

orientation: $\cancel{\theta}$ ... But if $\theta = 2\pi - \epsilon$, for small rotation, $\theta \to 0$, discontinuous

→ specify unit vectors for x_A^W, y_A^W ⟹ $\boxed{x_A^W = \text{coord's of } x_A \text{ axis w.r.t Frame } W.}$



} robot

world ref. frame

$$x_A^W = \begin{bmatrix} x_A \cdot x_W \\ x_A \cdot y_W \end{bmatrix}$$

$$y_A^W = \begin{bmatrix} y_A \cdot x_W \\ y_A \cdot y_W \end{bmatrix}$$

generalizes to 3D

Package these into a matrix

$$R_A^W = \begin{bmatrix} x_A \cdot x_W & y_A \cdot x_W \\ x_A \cdot y_W & y_A \cdot y_W \end{bmatrix} = \text{rotation matrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Specific to 2D

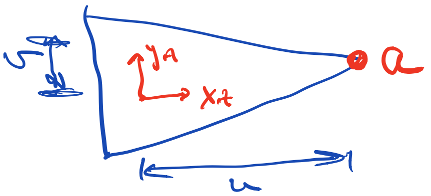
Given $q = (R, d)$, compute $\underbrace{\underset{\sim}{a}(q)}$ for some point $\underset{\sim}{a}$ on robot.

R: rot matrix

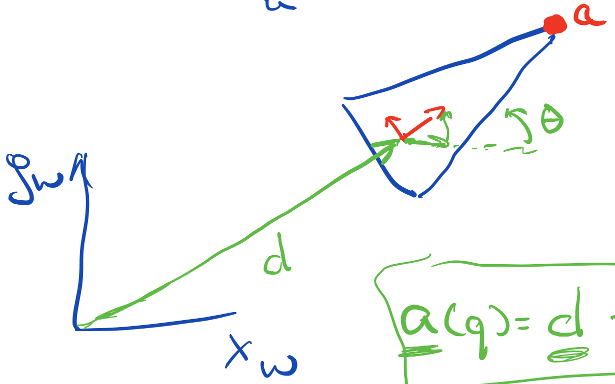
d: displacement

Location of a point $\underset{\sim}{a}$ on the robot, when robot is in configuration q.

Ex polygon robot



$a(q)$ for $q = \left(\begin{bmatrix} c\theta & -s\theta \\ s\theta & c\theta \end{bmatrix}, \begin{bmatrix} d_x \\ d_y \end{bmatrix} \right)$



For $q_0 = \left(I, \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right)$, $a(q_0) = \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{\text{local coord's for a in robot frame.}}$

$$\boxed{a(q) = d + u x_A + v y_A}$$

$$a(q) = d + \begin{bmatrix} u x_A \cdot x_W + v y_A \cdot x_W \\ u x_A \cdot y_W + v y_A \cdot y_W \end{bmatrix}$$

$$a(q) = d + \underbrace{\begin{bmatrix} x_A \cdot x_W & y_A \cdot x_W \\ x_A \cdot y_W & y_A \cdot y_W \end{bmatrix}}_{R_A^W} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{a(q_0)}$$

$$\underbrace{a(q) = d + R_A^W a(q_0)}_{\text{Forward Kinematic Map}} \Leftarrow \text{works in 2D and in 3D !}$$

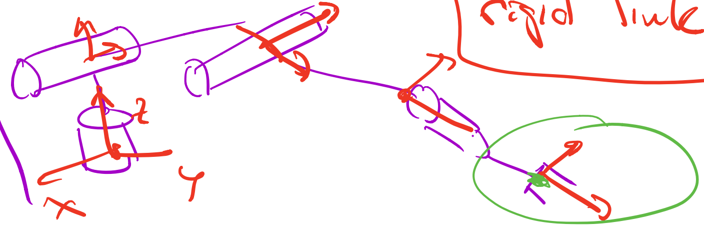
3D ~~rotations~~ transformations

$$R^W_A = \begin{bmatrix} x_A \cdot x_W & y_A \cdot x_W & z_A \cdot x_W \\ x_A \cdot y_W & y_A \cdot y_W & z_A \cdot y_W \\ x_A \cdot z_W & y_A \cdot z_W & z_A \cdot z_W \end{bmatrix}$$

$$d \in \mathbb{R}^3$$

Aside

1 frame per rigid link



$$q = (\theta_1, \theta_2, \cdots \theta_n)$$

About rotation Matrices

$f(\theta) \longleftrightarrow \underbrace{(R, d)}_{\text{End-effector frame}}$

- R is orthogonal

$$\begin{cases} \cdot\ r_i \cdot r_j = 0 & i \neq j \\ = 1 & i = j \\ \cdot\ c_i \cdot c_j = 0 & i \neq j \\ = 1 & i = j \end{cases} \Longrightarrow R R^T = I, \text{ or } \underline{\underline{R^T = R^{-1}}}$$

- $\det R = \underbrace{+1}_{\text{right handed coord sys: } \underbrace{x \times y = z}_{\text{right-hand rule}}} \Longrightarrow$ special

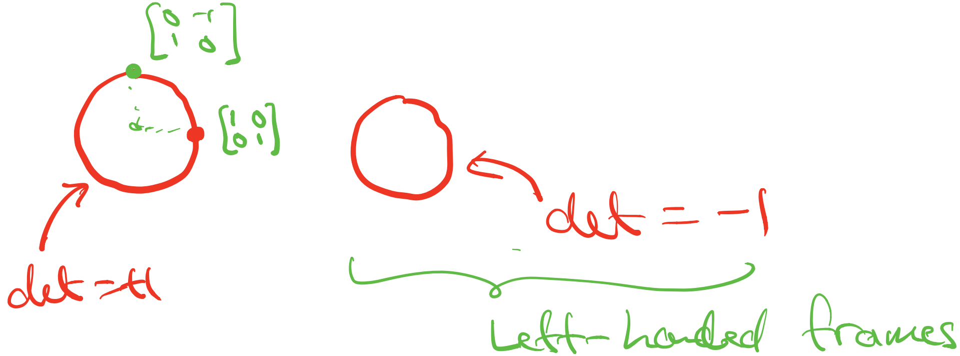
$\Longrightarrow$ special orthogonal Matrices, $\underline{\underline{SO}}(n)$ ↳ 2, 3

About $SO(2)$ ... $R \in \mathbb{R}^4$ but 3 constraints

$$c_1 \cdot c_1 = 1$$
$$c_1 \cdot c_2 = 0$$
$$c_2 \cdot c_2 = 1$$

Apply these constraints

$\mathbb{R}^4$



$$\underline{(R, d) \in SE(n)} \quad \text{for} \begin{cases} R \in SO(n) \\ d \in \mathbb{R}^n \end{cases}$$

↳ special Euclidean Group $SE(n)$

Often we represent $SE(n)$ by

$$\left[\begin{array}{c|c} R & d \\ 0 & 1 \end{array}\right] \leftarrow \text{Homogeneous Transformation Matrix}$$